Shape Deformation via Interior RBF

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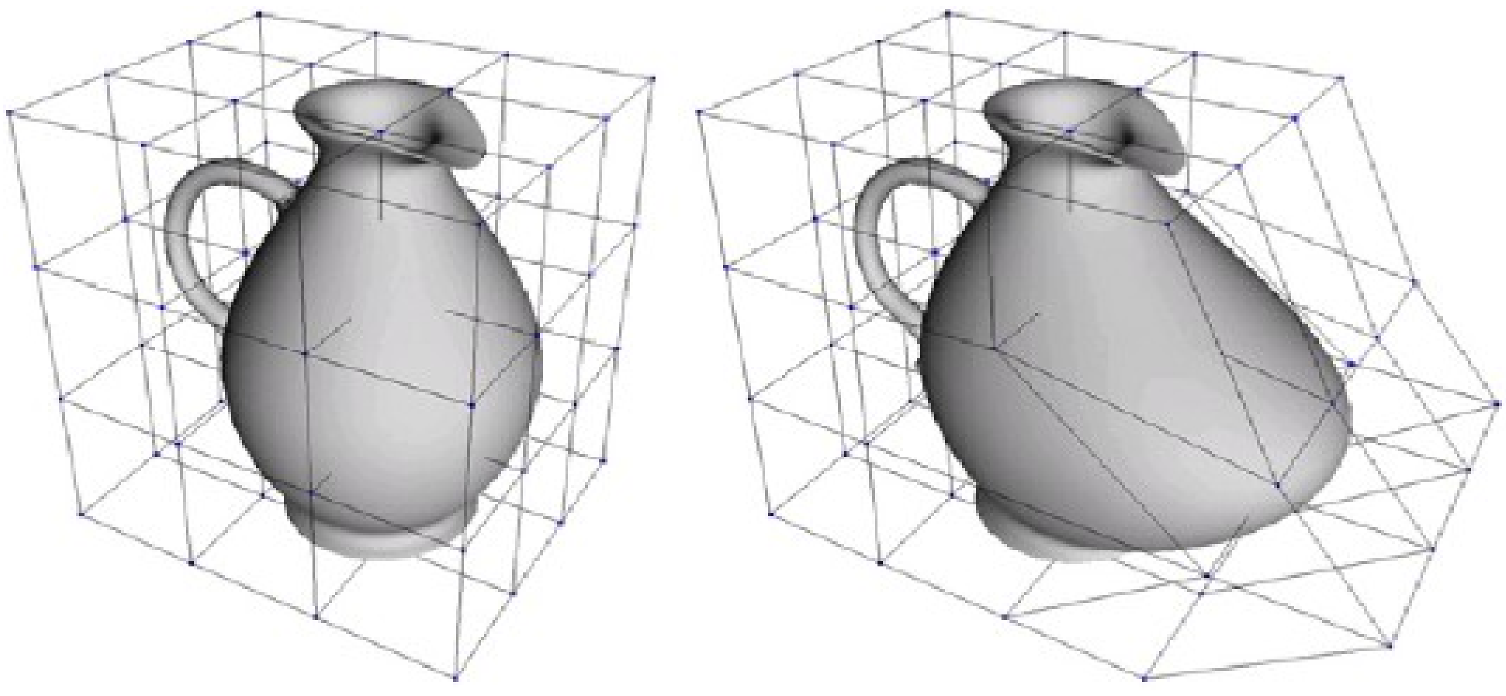
Motivation

- Animation tool for creating natural poses
- Space deformation vs. direct deformation
  - Subspace mapping: Can embed any type of object
  - Performance

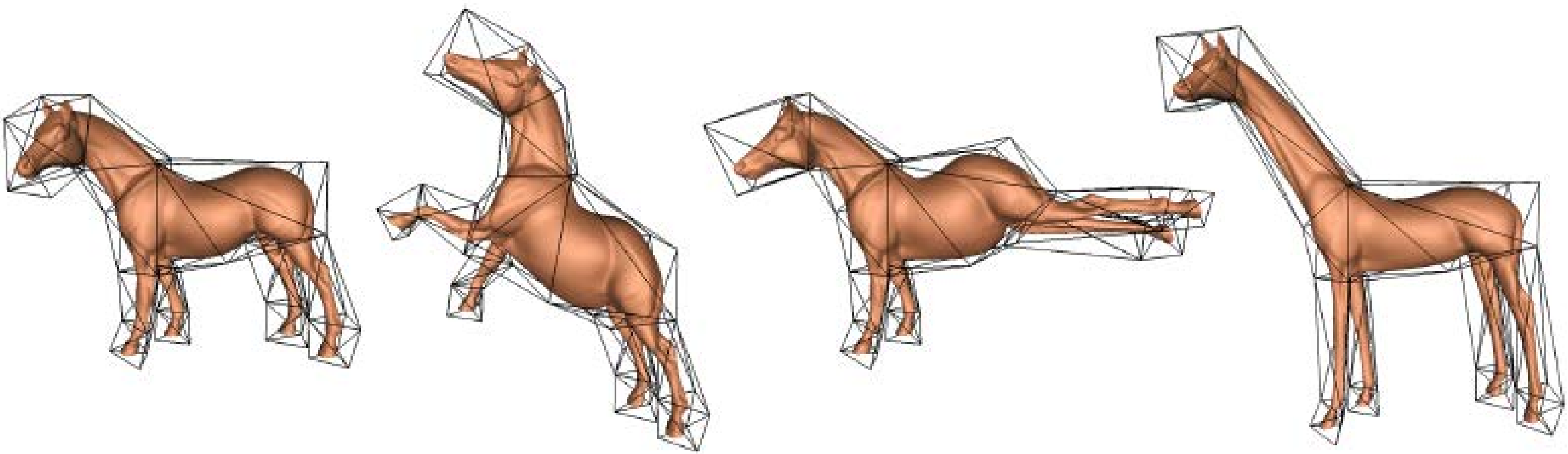
Control Structures

- Lattice [Sederberg and Parry 1986]

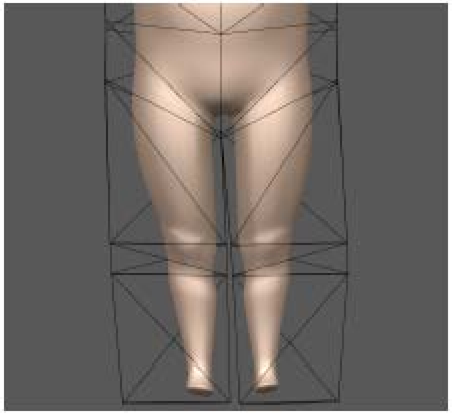


Cage-based Methods

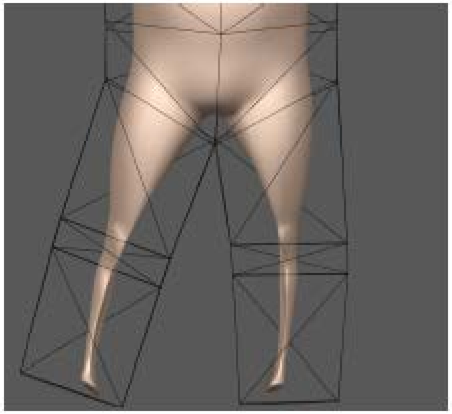
- Mean Value Coordinates [Ju et al. 2005]



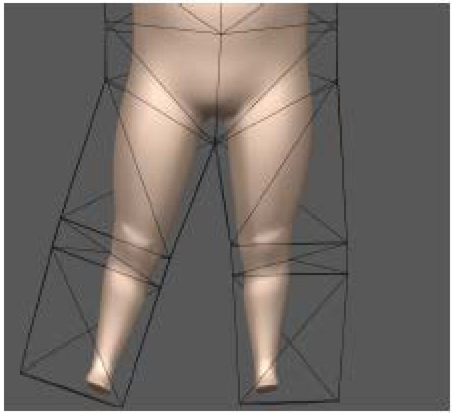
- Harmonic Coordinates [Joshi et al. 2007]
  - Interior locality property



Source

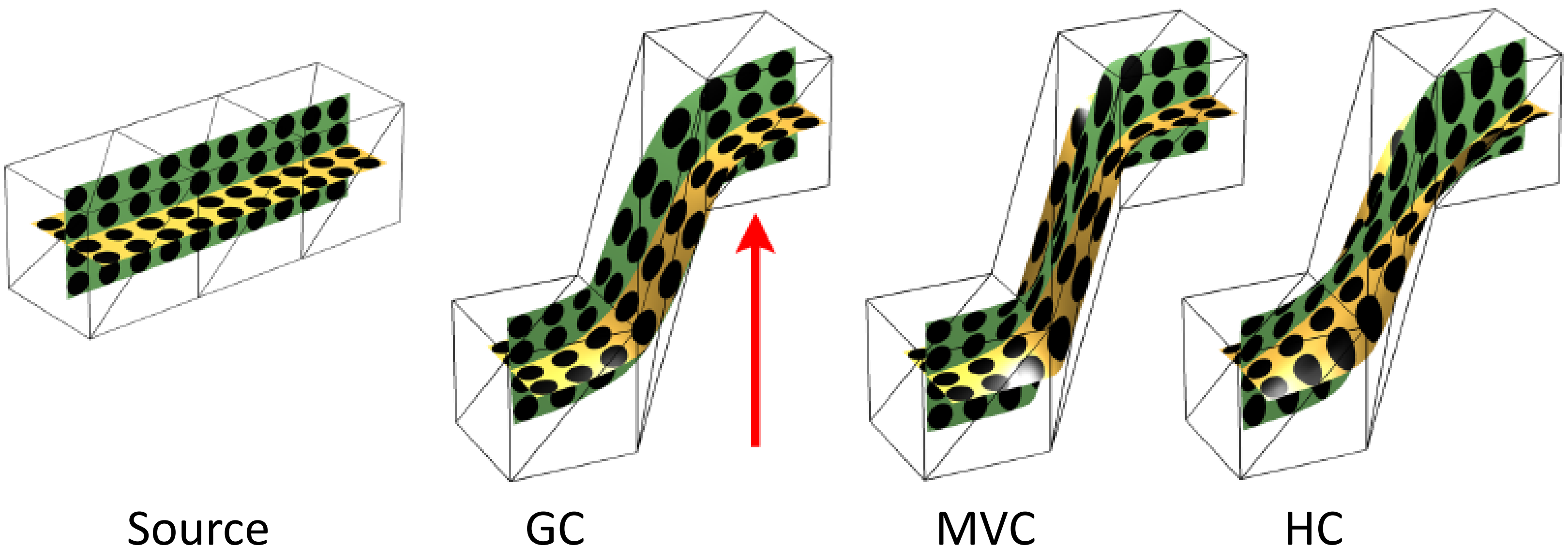


MVC

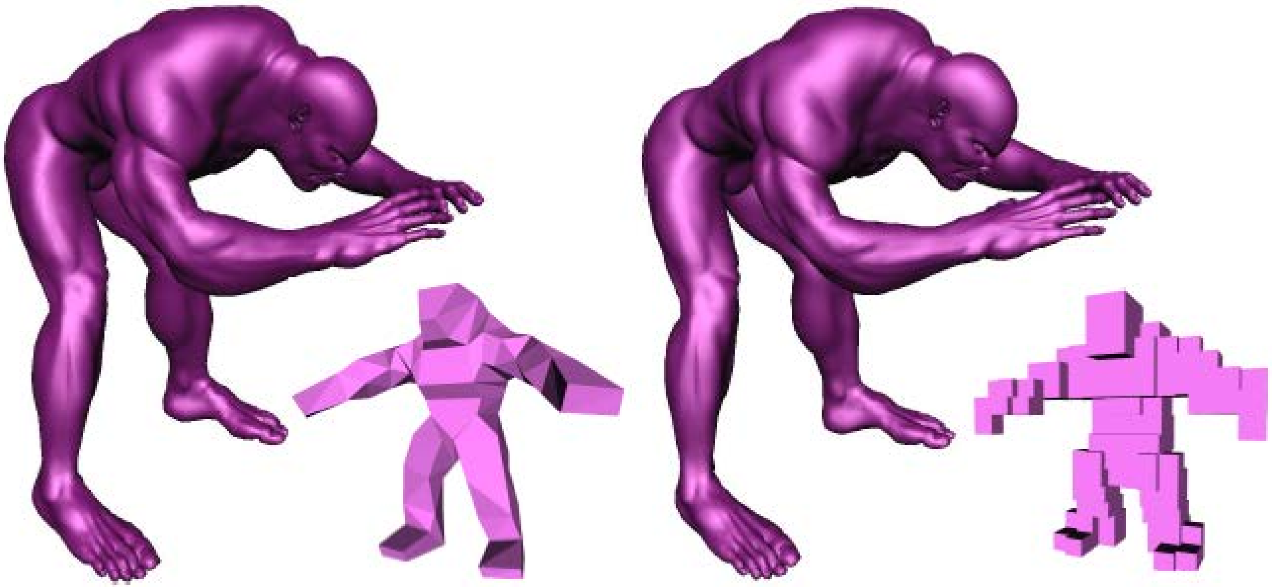


HC

- Green Coordinates [Lipman et al. 2008]
  - Quasi-conformal mapping

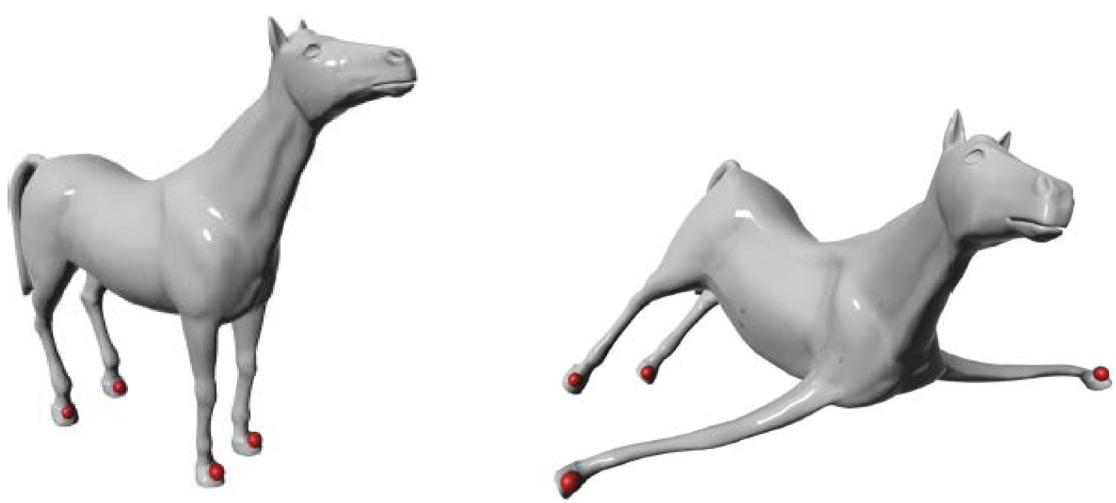


- VHM [Ben-Chen et al. 2009]
  - Convenient handles to control the deformation
  - Minimizes As-Rigid-As-Possible energy
  - The cage is still a problem: A well-constructed cage usually involves a considerable amount of manual work.



IRBF Properties

- Real-time
- Convenient handles to control the deformation
- Minimizes As-Rigid-As-Possible energy
- **No cage**
- Interior locality property



- Simpler formulation
- Can handle touching surface

Method Overview

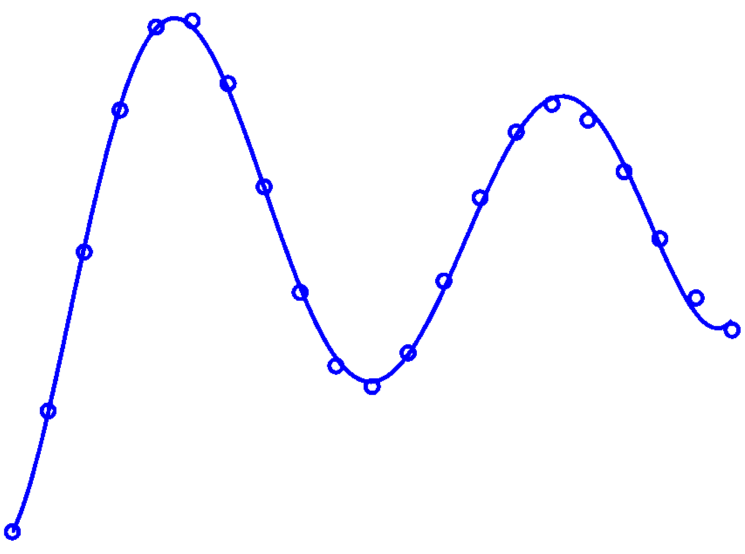
1. IRBF centers are sampled from the surface of the shape.
2. The shape is automatically filled with spheres.
3. Interior distances are calculated from the IRBF centers to:
   – The local rigidity structures that represent the spheres
   – The deformed points
   – The anchor points
4. Using *local/global* optimization the coefficients of the IRBF are calculated, and the shape is deformed.

RBF

- Scattered data approximation to functions on $\mathbb{R}^d$ [Franke 1982].
- An approximation at p:

$$F(p) = \sum_{c \in C} a_c \Phi(\| p - c \|), \quad a_c \in \mathbb{R},$$

where $C \subset \mathbb{R}^d$ is a set of centers and Φ is a real function.



IRBF

- Using interior distances d_I:

$$T_{\Pi}(p) = \underbrace{Ap + t}_{\text{Reproduce affine mapping}} + \sum_{c \in C} a_c \phi(p, c), \quad a_c \in \mathbb{R}^3$$

- where

$$\phi(p, c) = \frac{1}{D_I(p)} \Phi(d_I(p, c))$$

$$D_I(p) = \sum_{c \in C} \Phi(d_I(p, c))$$

Normalization for reproducing the constant function

$$\Phi(r) = \frac{1}{\sqrt{r^2 + h^2}}$$

Inverse multiquadric – regular everywhere

The Discrete Energy

- Energy:

Distortion　　　Regularization

$$E(\Pi) = \bar{U}(\Pi) + \lambda V(\mathbf{a})$$

Subject to positional constraints

  - Regularization term:

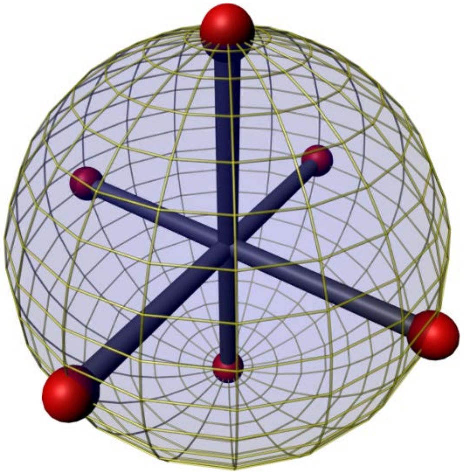
$$V(\mathbf{a}) = \sum_{c \in C} \| a_c \|^2$$

– As-Rigid-As-Possible [Sorkine and Alexa 2007; Chao et al. 2010] distortion measure for a set of spheres B:

$$\bar{U}(\Pi) = \sum_{b \in B} \bar{\rho}(b)$$

$$\bar{\rho}(b) = \sum_{i=1}^{3} w_{i,b} \, \| M_b e_{i,b} - e'_{i,b} \|^2, \;\; M_b \in SO(3)$$

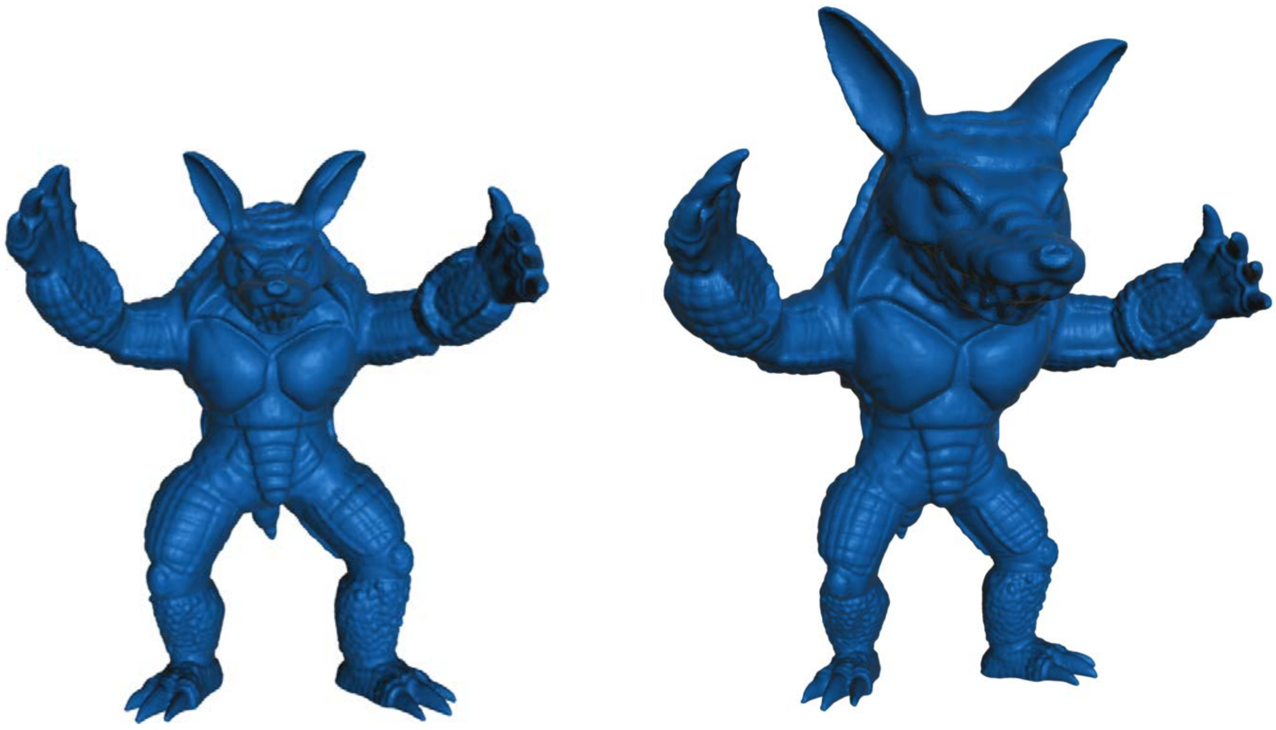
Source edges Target edges

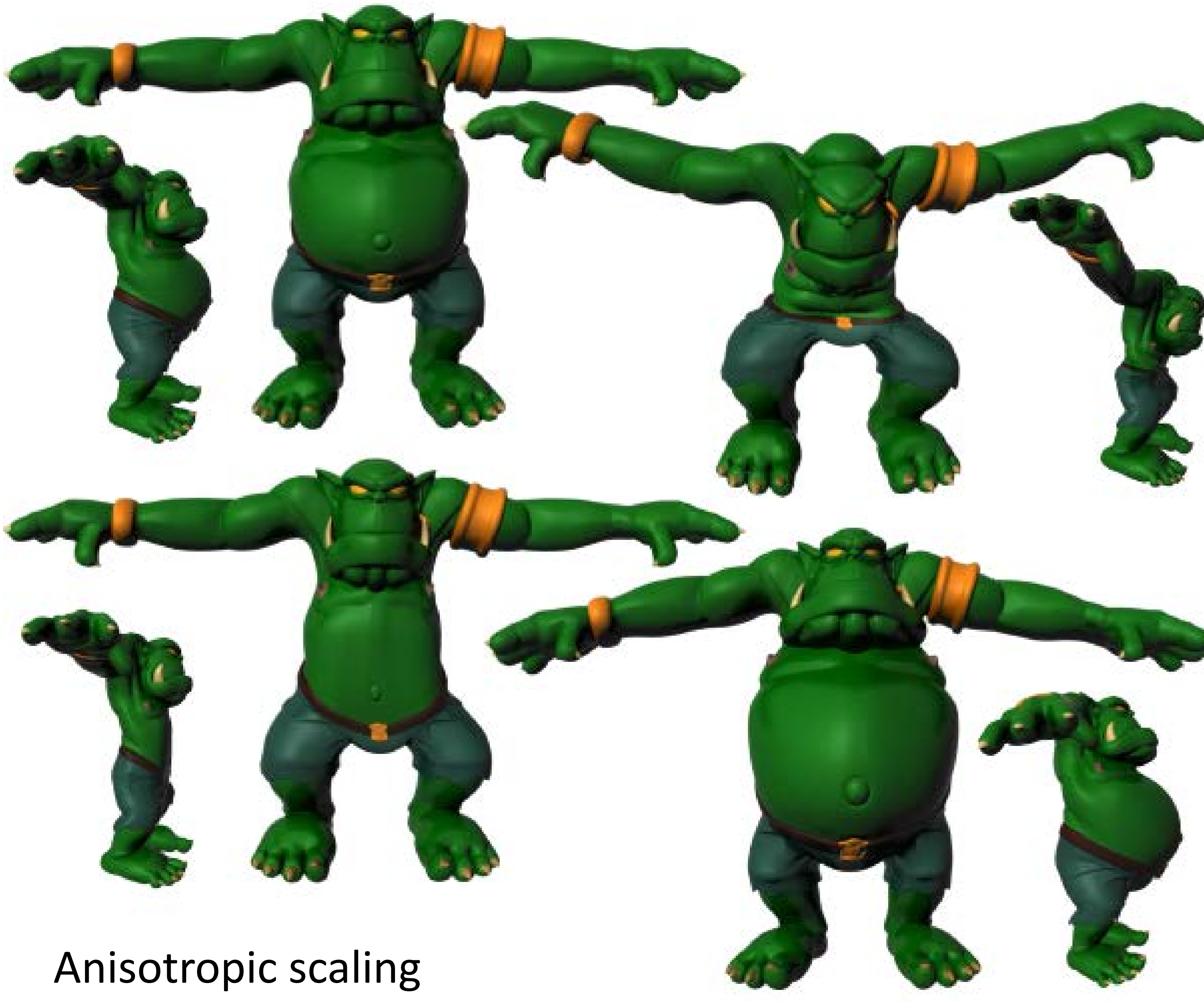


Local Shape and Volume Control

- Using prescribed transformation $S(b)$:

$$\overline{\rho}(b) = \sum_i \| M_b S(e_{i,b}) - e'_{i,b} \|^2$$

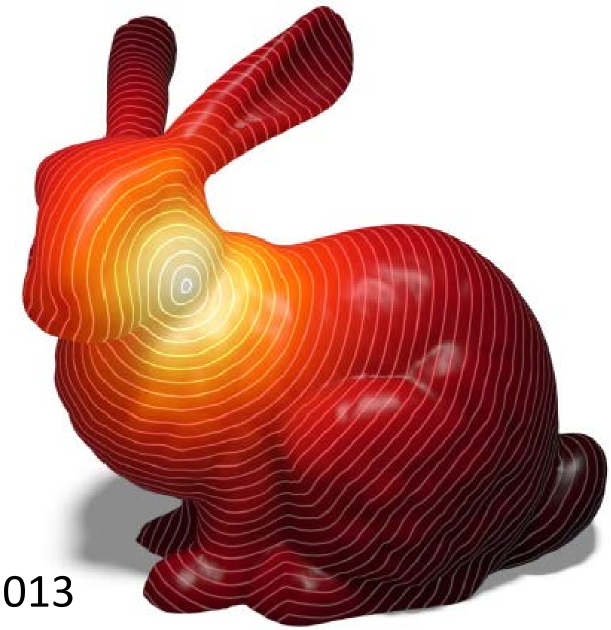




Anisotropic scaling

Interior Distances

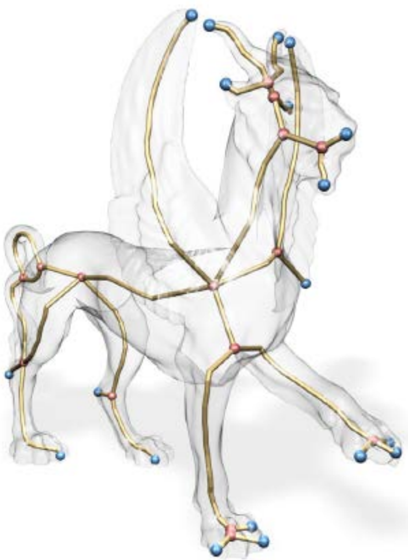
- Euclidean distances
- Fast Marching Method (FMM)
- Mean Value Coordinates (MVC)
- Geodesics in Heat [Crane et al. 2013]



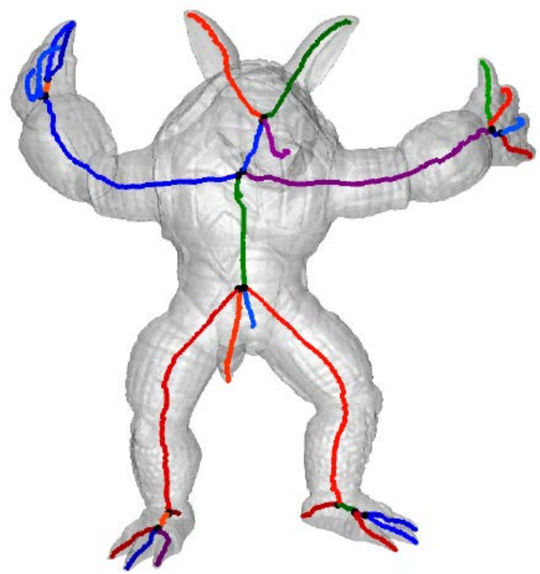
Crane et al. 2013

Setting The Controlling Spheres

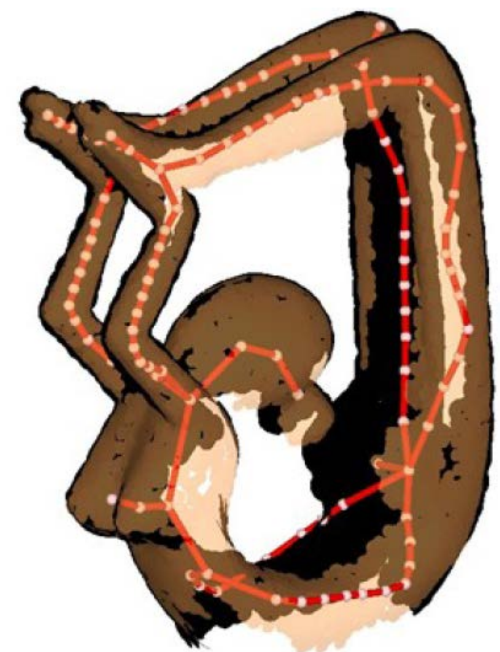
- Curve skeleton
  - Skeleton: [Au et al. 2008; Dey and Sun 2006] for meshes, and [Cao et al. 2010] for point clouds.
  - At sampled points along the skeleton, we place spheres with a radius of 90% of the distance from the sampled point to the surface.



Au et al. 2008

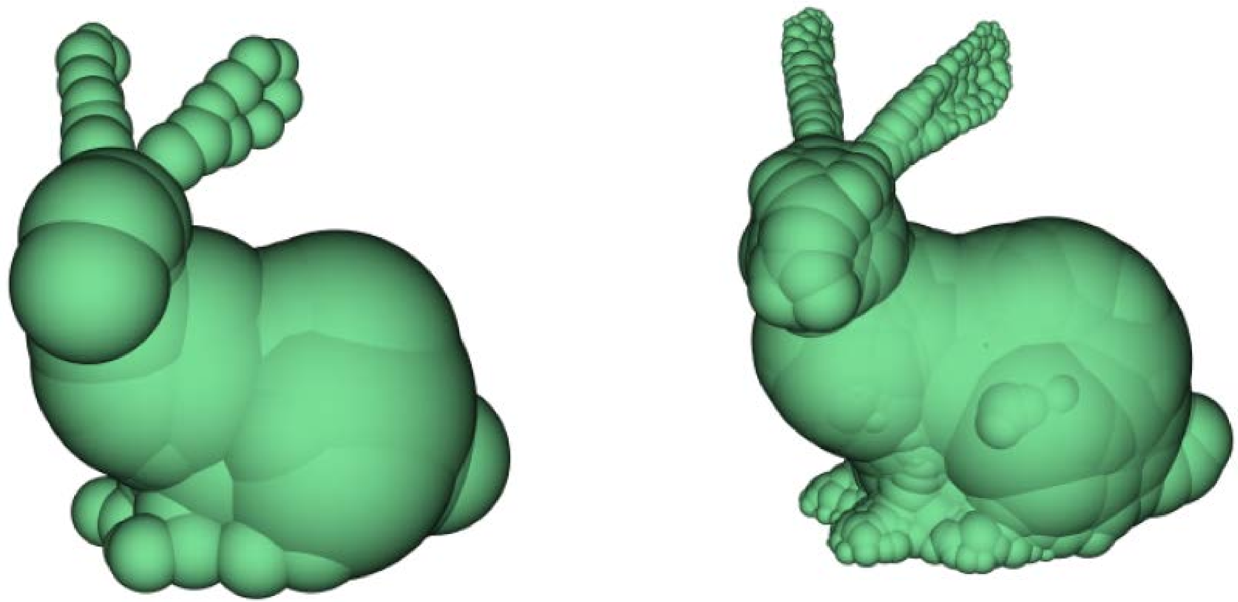


Dey and Sun 2006



Cao et al. 2010

- Sphere tree
  - Approximating a shape using spheres [Bradshaw and O'Sullivan 2004]



Bradshaw and O'Sullivan 2004

- Post processing: The spheres are pruned according to a threshold of maximum overlap, and minimal size.

Choosing The IRBF Centers

- Random sampling
- Farthest point sampling
- Sampling can be adaptive according to user hints (much easier than adapting a cage).

Algorithm Steps

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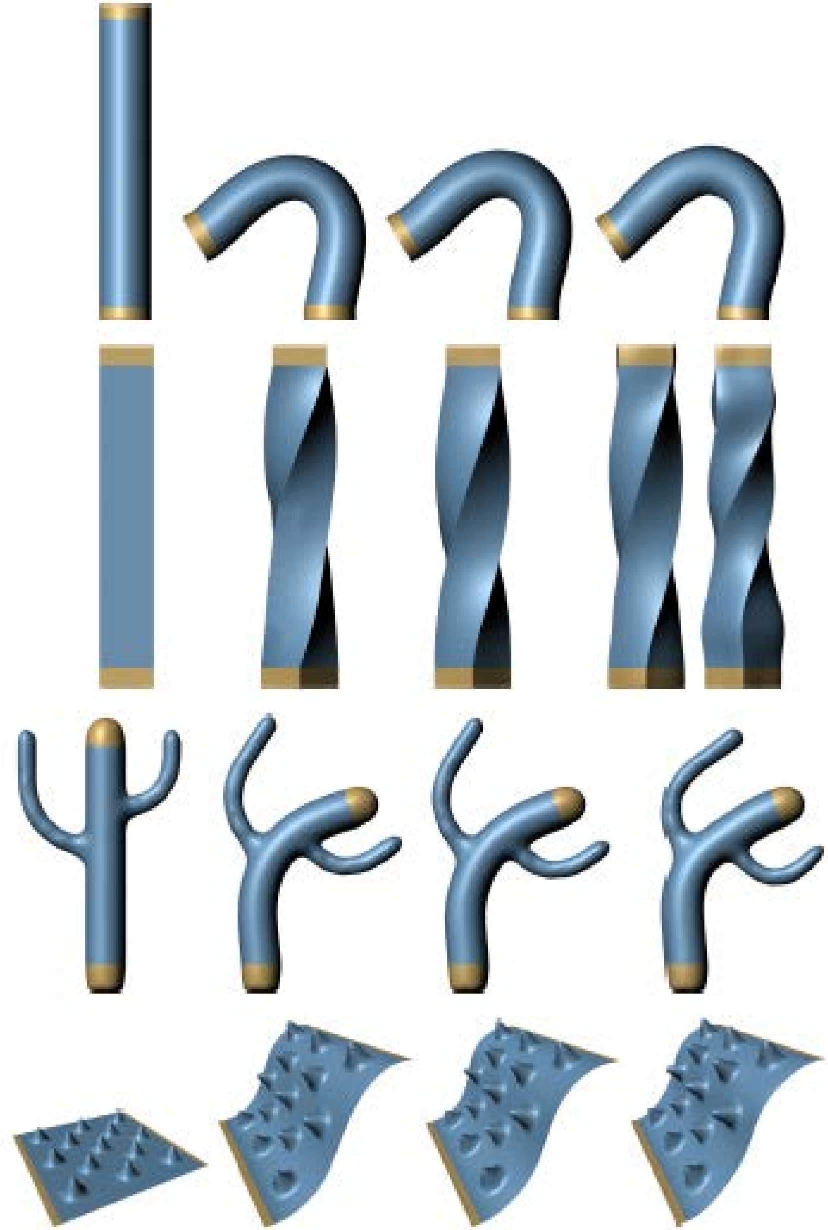
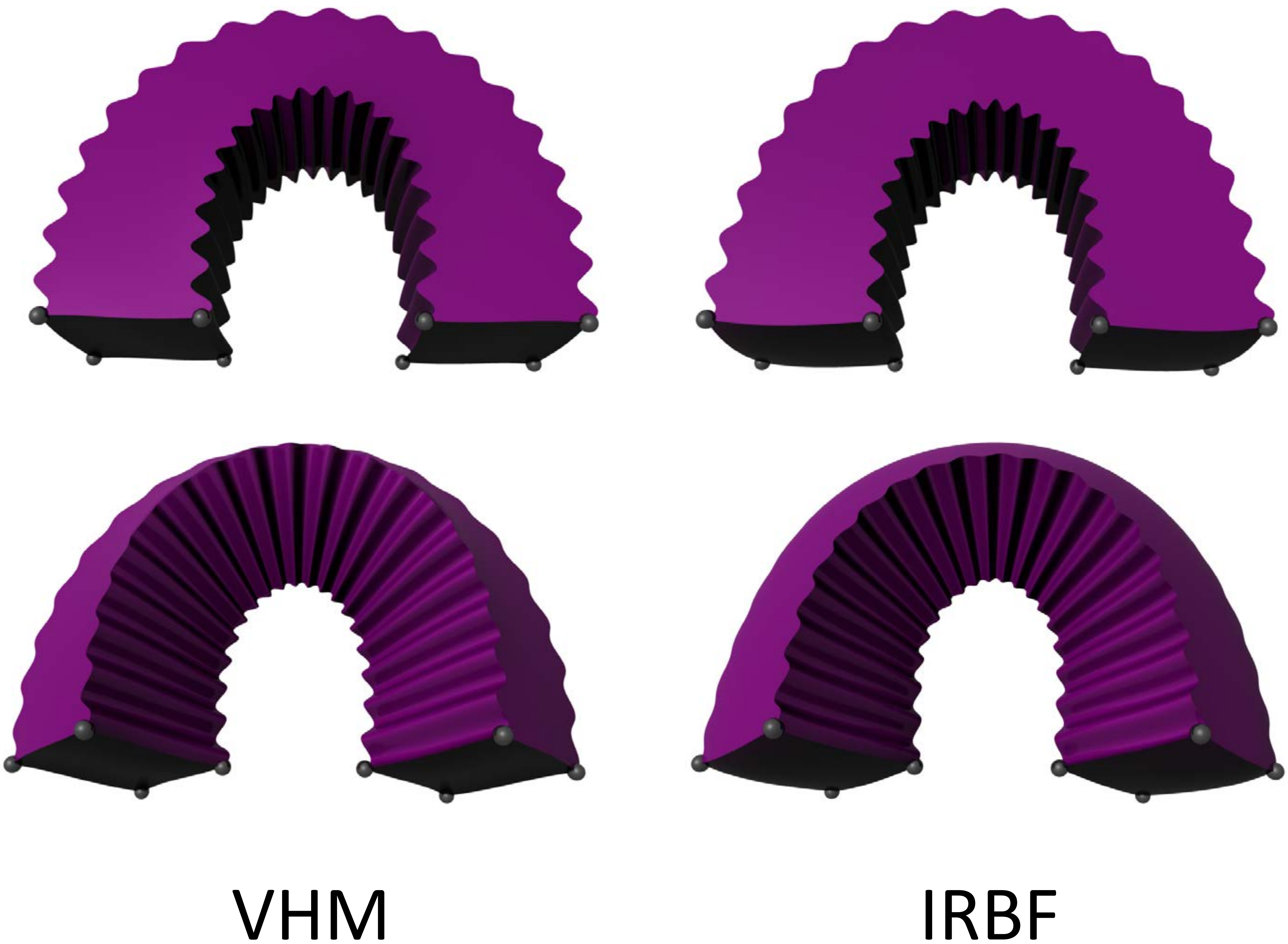
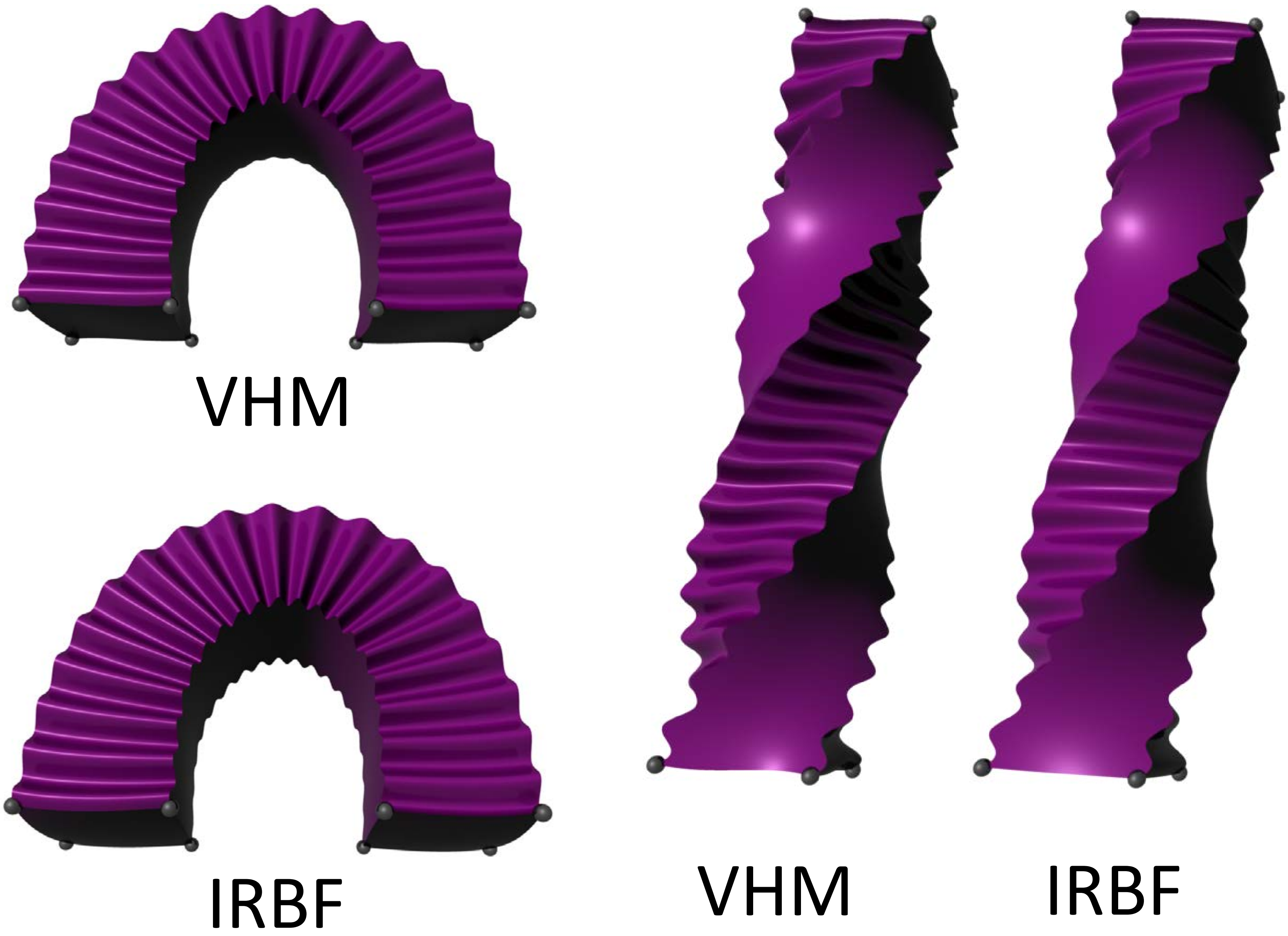
Results

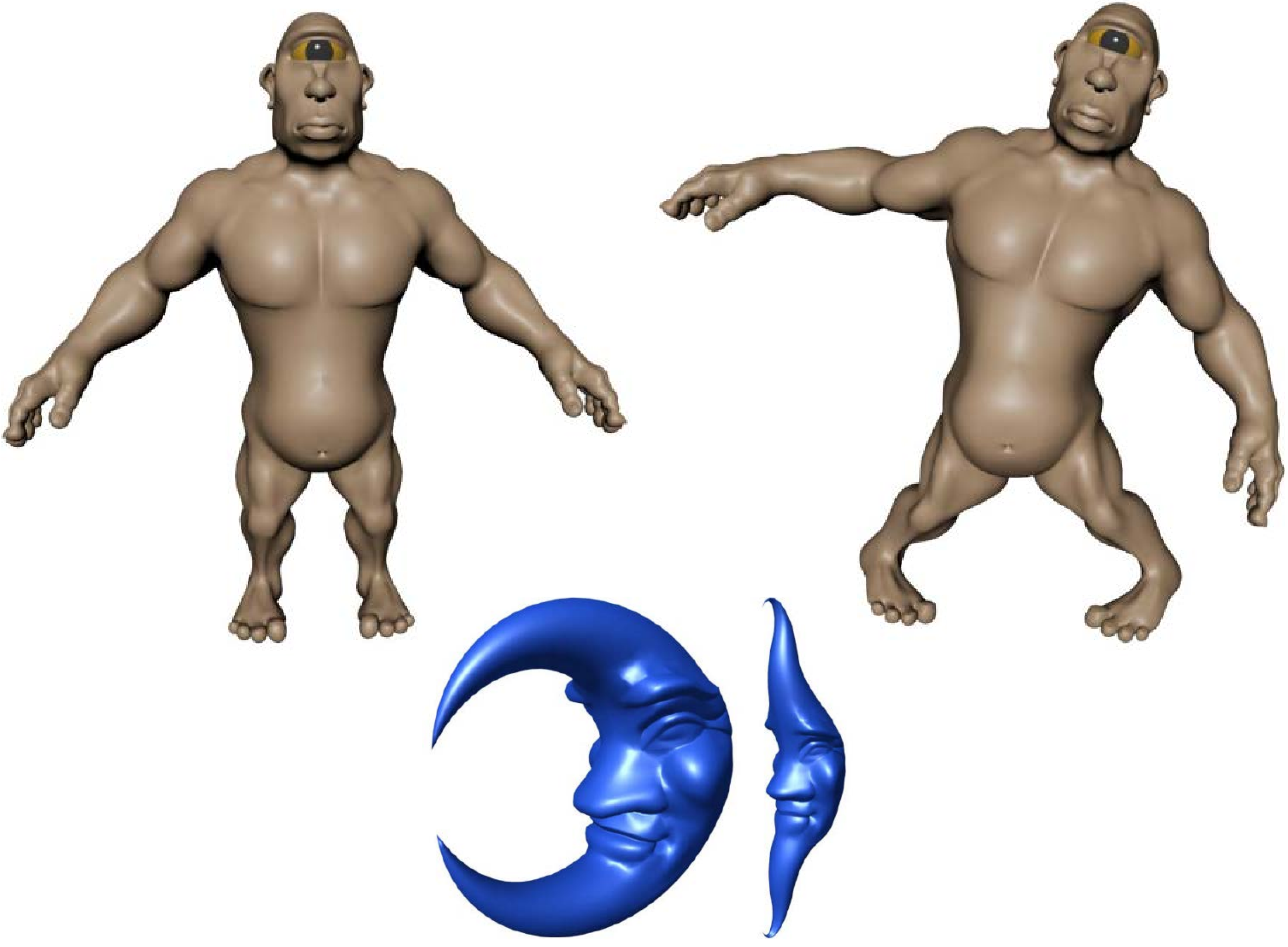
Survey models
[Botsch and Sorkine 2008]

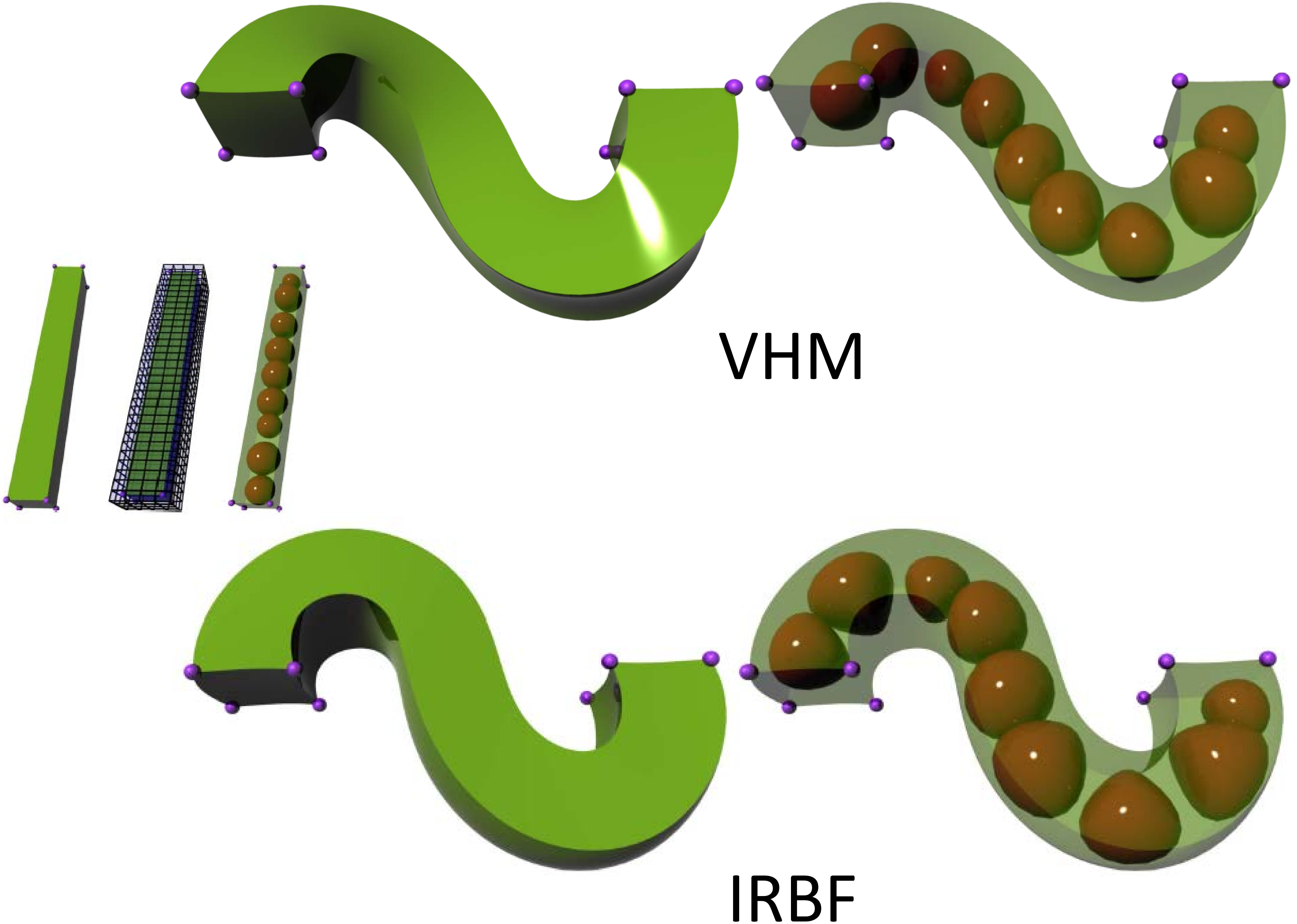


Source PriMo VHM IRBF


VHM
IRBF


VHM
IRBF
VHM
IRBF




VHM
IRBF